INSTITUTO FEDERAL DE EDUCAÇÃO, CIÊNCIA E TECNOLOGIA SUL-RIO-GRANDENSE Campus Passo Fundo

Engenharia Civil e Engenharia Mecânica

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Exercícios

Resolver as seguintes integrais por partes

a. $\int x\,sen(x)dx$

b. $\int \ln(x)dx$

c. $\int xe^{x}dx$

d. $\int x^{2}e^{x}dx$

e. $\int x\cos(x)dx$

f. $\int x^{2}e^{3x}dx$

g. $\int x\,sen(5x)dx$

h. $\int x^{3}e^{2x}dx$

i. $\int x\cos(3x)dx$

j. $\int xe^{-x}dx$

k. $\int \frac{\ln(x)}{\sqrt{x}}dx$

l. $\int \ln(x)x^{-3}dx$

m. $\int e^{x}\cdot\cos x\ dx$

n. $\int e^{2x}\cdot sen(3x)\ dx$

Respostas

a. -xcos(x) + sen(x) + c

b. xln(x) – x + c

c. $xe^{x} - e^{x} + c$

d. $x^{2}e^{x} - 2xe^{x} + 2e^{x} + c$

e. xsen(x) + cos(x) + c

f. $e^{3x}(\frac{x^{2}}{3} - \frac{2x}{9} + \frac{2}{27}) + c$

g. $\frac{-x}{5}\cos(5x) + \frac{1}{25}sen(5x) + c$

h. $e^{2x}(\frac{x^{3}}{2} - \frac{3x^{2}}{4} + \frac{3x}{4} - \frac{3}{8}) + c$

Resolva as seguintes integrais de potências trigonométricas.

1. $\int sen^{3}xdx$
2. $\int sen^{4}x\,.\cos x dx$
3. $\int \cos^{2}(\frac{x}{2})dx$
4. $\int sen^{2}x.\cos^{3}xdx$
5. $\int sen^{2}(3x).\cos^{2}(3x)dx$
6. $\int \frac{\cos^{3}(3x)}{\sqrt[3]{sen(3x)}}dx$

Respostas

1. $-\cos(x) + \frac{\cos^{3}(x)}{3} + c$
2. $\frac{sen^{5}(x)}{5} + c$
3. $\frac{x}{2} + \frac{1}{2}sen(x) + c$
4. $\frac{sen^{3}(x)}{3} - \frac{sen^{5}(x)}{5} + c$
5. $\frac{x}{8} - \frac{sen(12x)}{96} + c$
6. $\frac{1}{2}\sqrt[3]{(sen(3x)^{2}} - \frac{1}{8}\sqrt[3]{(sen(3x))^{8}} + c$

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Resolva as seguintes integrais por substituição trigonométrica.

a) $\int \frac{dx}{x^2\sqrt{16-x^2}}$

b) $\int \frac{\sqrt{9-x^2}}{x^2}dx$

c) $\int \frac{dt}{t^3\sqrt{t^2-25}}$

d) $\int \sqrt{x^2+5}dx$

e) $\int \frac{dx}{x^3\sqrt{x^2-9}}$

f) $\int \frac{\sqrt{x^2-9}}{x}dx$

g) $\int \frac{1}{\sqrt{4+x^2}}dx$

h) $\int \frac{1}{(4x^2+9)^2}dx$

i) $\int \frac{x^2}{1+x^2}dx$

j) $\int \frac{1}{\sqrt{4-9x^2}}dx$

Respostas

a) $\frac{-\sqrt{16-x^2}}{16x}+c$

b) $\frac{-\sqrt{9-x^2}}{x}-\text{arcsen}(\frac{x}{3})+c$

c) $\frac{1}{250}(\text{arc sec}\frac{t}{5}+\frac{5\sqrt{t^2-25}}{t^2})+c$

d) $\frac{x\sqrt{x^2+5}}{2}+\frac{5}{2}\ln\left|\frac{\sqrt{x^2+5}}{\sqrt{5}}+\frac{x}{\sqrt{5}}\right|+c$

e) $\frac{1}{54}\text{arc sec}\frac{x}{3}+\frac{\sqrt{x^2-9}}{18x^2}+c$

f) $3\left(\frac{\sqrt{x^2-9}}{3}-arc\sec\left(\frac{x}{3}\right)\right)+c$

g) $\ell n\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right|+C$

h) $\frac{1}{54}arctg\left(\frac{2x}{3}\right)+c$

i) $x-arctg(x)+c$

j) $\frac{1}{3}arcsen\left(\frac{3x}{2}\right)+c$

Resolva as integrais das seguintes funções racionais – Caso 1 e Caso 2

1) $\int \frac{3x+13}{(x-4)(x+10)}dx$

2) $\int \frac{dx}{x(x+1)}$

3) $\int \frac{x^2-16x-11}{(x-3)(x+2)^2}dx$

4) $\int \frac{1}{x^2-4}dx$

5) $\int \frac{x}{(x+3)^2}dx$

6) $\int \frac{dx}{x^2+x-2}$

7) $\int \frac{(x-1)}{x^3-x^2-2x}dx$

8) $\int \frac{x^3-1}{x^2(x-2)^3}dx$

9) $\int \frac{x^3+1}{x(x+4)}dx$

10) $\int \frac{x^3-1}{x^2-x-2}dx$

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Respostas

1) $\frac{25}{14}\ln|x-4|+\frac{17}{14}\ln|x+10|+c$

2) $\ln|x|-\ln|x+1|+c$

3) $-2\ln|x-3|+\frac{5}{x+2}+3\ln|x+2|+c$

4) $\frac{-1}{4}\ln|x+2|+\frac{1}{4}\ln|x-2|+c$

5) $\frac{3}{x+3}+\ln|x+3|+c$

6) $\frac{1}{3}\left[\ln|x-1|-\ln|x+2|\right]+C$

7) $\frac{1}{2}\ln|x|+\frac{1}{6}\ln|x-2|-\frac{2}{3}\ln|x+1|+c$

8) $\frac{-1}{8x}+\frac{3}{16}\ln(x)-\frac{7}{8(x-2)^2}-\frac{5}{4(x-2)}-\frac{3}{16}\ln(x-2)+c$

Ou simplificando

$$\frac{-11x^2+17x-4}{8x(x-2)^2}+\frac{3}{16}\ln\left|\frac{x}{x-2}\right|+c$$

9) $\frac{1}{4}\ln|x|+\frac{63}{4}\ln|x+4|+\frac{x^2}{2}-4x+c$

10) $\frac{x^2}{2}+x+\frac{2}{3}\ln|x+1|+\frac{7}{3}\ln|x-2|+c$