

Engenharia Civil

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Exercícios

Resolver as seguintes integrais por partes

a. $\int x \sin(x) dx$

b. $\int \ln(x) dx$

c. $\int x e^x dx$

d. $\int x^2 e^x dx$

e. $\int x \cos(x) dx$

f. $\int x^2 e^{3x} dx$

g. $\int x \sin(5x) dx$

h. $\int x^3 e^{2x} dx$

i. $\int x \cos(3x) dx$

j. $\int x e^{-x} dx$

k. $\int \frac{\ln(x)}{\sqrt{x}} dx$

l. $\int \ln(x) x^{-3} dx$

m. $\int e^x \cdot \cos x dx$

n. $\int e^{2x} \cdot \sin(3x) dx$

Respostas

a. $-x \cos(x) + \sin(x) + c$

b. $x \ln(x) - x + c$

c. $x e^x - e^x + c$

d. $x^2 e^x - 2x e^x + 2e^x + c$

e. $x \sin(x) + \cos(x) + c$

f. $e^{3x} \left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} \right) + c$

g. $\frac{-x}{5} \cos(5x) + \frac{1}{25} \sin(5x) + c$

h. $e^{2x} \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right) + c$



Resolva as seguintes integrais de potências trigonométricas.

1. $\int \sin^3 x dx$
2. $\int \sin^4 x \cdot \cos x dx$
3. $\int \cos^2\left(\frac{x}{2}\right) dx$
4. $\int \sin^2 x \cdot \cos^3 x dx$
5. $\int \sin^2(3x) \cdot \cos^2(3x) dx$
6. $\int \frac{\cos^3(3x)}{\sqrt[3]{\sin(3x)}} dx$

Respostas

1. $-\cos(x) + \frac{\cos^3(x)}{3} + c$
2. $\frac{\sin^5(x)}{5} + c$
3. $\frac{x}{2} + \frac{1}{2}\sin(x) + c$
4. $\frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + c$
5. $\frac{x}{8} - \frac{\sin(12x)}{96} + c$
6. $\frac{1}{2}\sqrt[3]{(\sin(3x))^2} - \frac{1}{8}\sqrt[3]{(\sin(3x))^8} + c$

Resolva as seguintes integrais por substituição trigonométrica.

- a) $\int \frac{dx}{x^2 \sqrt{16-x^2}}$
- b) $\int \frac{\sqrt{9-x^2}}{x^2} dx$
- c) $\int \frac{dt}{t^3 \sqrt{t^2-25}}$
- d) $\int \sqrt{x^2+5} dx$
- e) $\int \frac{dx}{x^3 \sqrt{x^2-9}}$
- f) $\int \frac{\sqrt{x^2-9}}{x} dx$
- g) $\int \frac{1}{\sqrt{4+x^2}} dx$
- h) $\int \frac{1}{(4x^2+9)^2} dx$
- i) $\int \frac{x^2}{1+x^2} dx$
- j) $\int \frac{1}{\sqrt{4-9x^2}} dx$

Respostas

- a) $\frac{-\sqrt{16-x^2}}{16x} + c$
- b) $\frac{-\sqrt{9-x^2}}{x} - \arcsen\left(\frac{x}{3}\right) + c$
- c) $\frac{1}{250} \left(\operatorname{arcsec} \frac{t}{5} + \frac{5\sqrt{t^2-25}}{t^2} \right) + c$
- d) $\frac{x\sqrt{x^2+5}}{2} + \frac{5}{2} \ln \left| \frac{\sqrt{x^2+5}}{\sqrt{5}} + \frac{x}{\sqrt{5}} \right| + c$
- e) $\frac{1}{54} \operatorname{arcsec} \frac{x}{3} + \frac{\sqrt{x^2-9}}{18x^2} + c$
- f) $3 \left(\frac{\sqrt{x^2-9}}{3} - \operatorname{arcsec} \left(\frac{x}{3} \right) \right) + c$
- g) $\ln \left| \frac{\sqrt{4+x^2}}{2} + \frac{x}{2} \right| + C$
- h) $\frac{1}{54} \operatorname{arctg} \left(\frac{2x}{3} \right) + c$
- i) $x - \operatorname{arctg}(x) + c$
- j) $\frac{1}{3} \operatorname{arcsen} \left(\frac{3x}{2} \right) + c$



Resolva as integrais das seguintes funções racionais – Caso 1 e Caso 2

1) $\int \frac{3x+13}{(x-4)(x+10)} dx$

2) $\int \frac{dx}{x(x+1)}$

3) $\int \frac{x^2-16x-11}{(x-3)(x+2)^2} dx$

4) $\int \frac{1}{x^2-4} dx$

5) $\int \frac{x}{(x+3)^2} dx$

6) $\int \frac{dx}{x^2+x-2}$

7) $\int \frac{(x-1)}{x^3-x^2-2x} dx$

8) $\int \frac{x^3-1}{x^2(x-2)^3} dx$

9) $\int \frac{x^3+1}{x(x+4)} dx$

10) $\int \frac{x^3-1}{x^2-x-2} dx$

Respostas

1) $\frac{25}{14} \ln|x-4| + \frac{17}{14} \ln|x+10| + c$

2) $\ln|x| - \ln|x+1| + c$

3) $-2 \ln|x-3| + \frac{5}{x+2} + 3 \ln|x+2| + c$

4) $\frac{-1}{4} \ln|x+2| + \frac{1}{4} \ln|x-2| + c$

5) $\frac{3}{x+3} + \ln|x+3| + c$

6) $\frac{1}{3} [\ln|x-1| - \ln|x+2|] + C$

7) $\frac{1}{2} \ln|x| + \frac{1}{6} \ln|x-2| - \frac{2}{3} \ln|x+1| + c$

8) $\frac{-1}{8x} + \frac{3}{16} \ln(x) - \frac{7}{8(x-2)^2} - \frac{5}{4(x-2)} - \frac{3}{16} \ln(x-2) + c$

Ou simplificando

$\frac{-11x^2+17x-4}{8x(x-2)^2} + \frac{3}{16} \ln \left| \frac{x}{x-2} \right| + c$

9) $\frac{1}{4} \ln|x| + \frac{63}{4} \ln|x+4| + \frac{x^2}{2} - 4x + c$

10) $\frac{x^2}{2} + x + \frac{2}{3} \ln|x+1| + \frac{7}{3} \ln|x-2| + c$